HSCP Maths Mash-up #3

No calculators, abaci, or props!

1. What is the value of $(1-(1-(1-1)))-(((1-1)-1)-1)$?

2. A knockout tennis tournament starts with 128 competitors. How many matches are required to determine the winner?

3. Five large gold ingots have a mean average weight of 12 kilograms. One is stolen, and the mean weight of the remaining ingots is 11 kg. What is the weight of the stolen ingot?

4. In Shakespeare's *As You Like It*, Rosalind speaks to Orlando about "He that will divide a minute into a thousand parts". One thousandth of a minute is how many seconds?

5. How many of the whole numbers 1, 12, 123, 1234, 12345, 123456 are prime?

6. Foo has three times as many llamas as lambs. Bar has twice as many lambs as llamas. They have seventeen animals in total. How many of them are llamas?

7. Only one of the following five statements is true. Which one?
   A. "B is true."
   B. "E is false."
   C. "All five statements are true."
   D. "All five statements are false."
   E. "A is false."

8. Three discs each have a positive whole number on both sides. They are tossed up and allowed to land on a flat surface. The sum of the three numbers showing could be any of the whole numbers from 3 to 10. What are the pairs of numbers on each disc?

9. Consider the *regular tetrahedron*, the triangular pyramid whose four sides are all identical equilateral triangles. In how many ways can it be orientated such that it has the same physical position overall but some or all of its faces swapped around?

10. Red, green, yellow, and blue balls are placed in a bag—five of each colour. Three balls are drawn from the bag at random and without putting them back in the bag. What is the probability that the three balls drawn all have different colours?

11. Project Z is usually assigned to eight workers and takes them fifteen hours to complete. On this occasion, it is hoped to complete Project Z a bit faster, so it is assigned to ten workers. But with one quarter of it remaining, some of its workers must be requisitioned for other projects. What is the minimum number of workers who must remain on Project Z for it still to be completed one hour faster than usual?

12. A *face diagonal* is a diagonal (a line between non-adjacent corners) of a face of a solid shape. How many face diagonals has a cuboid? What is the length of any face diagonal of the *unit cube* (the cube with edges of length 1 unit)?

13. Foo, Bar, and Qux all have 24-hour digital watches. Foo's watch runs at the right speed but Bar's loses ten minutes every day and Qux's loses fifteen minutes every day. At noon on the 1st of January, all three watches show the right time. On what date will they all show the right time again?

14. I drive from London to Brighton then back again on the same route in reverse. On the way to Brighton my average speed is 40 miles per hour. On the way back it is 60 mph. What is my average speed for both journeys considered together?

15. A large rectangle is drawn on a blank wall and divided up into three columns and three rows. Every cell in the resultant grid is painted a distinct colour. In how many different ways can the grid be painted without changing the *relative* cell-positions of the colours?

16. A regular hexagon, with sides of length 1 unit, has its corners labelled P,Q,R,S,T,U, in clockwise order. What are the precise lengths of the diagonals PS and PR?

17. What is the value of $1234567897 \times 1234567898 - 1234567896 \times 1234567899$?

18. Of all the whole numbers from 1 to 6000, how many are *not* multiples of 2, 3, or 5?

19. Every corner of a regular pentagon is connected to every other by a straight line (resulting in a *pentagonal mystic rose*, i.e., a regular pentagon inscribed by a regular *pentagram*). How many triangles are there in the figure?

20. In chess, the knight's move is the familiar $\{2,1\}$-L-shaped jump. The *knight power* of a given square on a given chessboard is defined as the number of squares a knight may move to from that square in one move on an otherwise empty example of that board. What is the mean average knight power of the squares on a standard (8-by-8) board?