

## HSCP Maths Mash-up #4

No calculators, abaci, or props!

1. What is the minimum number of people that must be present in a (large) room such that at least two people in the room share the same birthday?

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2. How many seconds are there in  $\frac{1}{4}$  of  $\frac{1}{6}$  of  $\frac{1}{8}$  of a day?

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3. A rectangle is divided up into four triangles by its diagonals. Either of the two triangles incident with a long side of the rectangle is labelled P, and either of the two triangles incident with a short side of the rectangle is labelled Q. What is the ratio of the area of P to the area of Q?

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4. A 3-by-3 number square contains the counting numbers 1 to 9 arranged like a typical telephone keypad. Choose three numbers from this square in such a way that no row or column is missed out, then multiply the three numbers together. By choosing the three numbers in this way, what is the largest possible product?

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5. The sum of five consecutive integers is 3505. What is the smallest of these integers?

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6. The length of the perimeter of a square piece of card is 20 units. The card is cut into two rectangles. The length of the perimeter of one of the rectangles is 16 units. What is the length of the perimeter of the other rectangle?

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7. If the fractions  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{5}$ ,  $\frac{4}{7}$ ,  $\frac{5}{9}$  are arranged in increasing order of size, which one is in the middle?

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8. A list is made of every digit which is the units digit of at least one prime number. How many digits are in the list?

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9. A *face diagonal* is a diagonal (a line between non-adjacent corners) of a face of a solid shape. How many face diagonals has a *regular dodecahedron*, the convex regular solid shape ("Platonic solid") whose sides are all identical regular pentagons?

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10. What name is given to any closed polygon having the fewest possible number of sides such that its sides are all the same length but its angles are not necessarily all the same size?

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11. Five bags each contain some marbles. The mean average number of marbles in a bag is 5, the median is 6, and the only mode is 2. Precisely how many marbles are in each bag?

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12. In a pile of 101 coins, all are unbiased except for one which has heads on both sides. A coin is chosen from the pile at random and tossed ten times. It comes up heads every time. What is the probability that it will come up heads on the eleventh toss as well?

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13. The positive whole number  $n$  has the property that the sum of itself and its digits is 313. What are the possible values of  $n$ ?

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14. In how many ways can a cube be orientated such that it has the same physical position overall but some or all of its faces swapped around?

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15. I made a car journey of 144 miles, stopping for one hour along the way. Had I travelled at an average speed, when moving, four miles per hour faster but stopped for half an hour longer, the journey would have had the same duration. What was my average speed (in mph) when moving?

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16. At the reading of late Great Grandpa Garply's will, Foo and Bar are told that they will inherit one sixth of the total estate between them, but each amount must be a *simple* fraction of the estate, i.e., a fraction with numerator 1. In how many different ways can this portion of late Great Grandpa Garply's estate be split between Foo and Bar?

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17. A square has its corners labelled P,Q,R,S, in clockwise order. It encloses two identical circular arcs which both connect corners Q and S, but one has centre P and the other has centre R. What proportion of the area of the square is enclosed between the two arcs?

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18. A sequence, whose 1st term is  $-5$  and 26th term is  $+2$ , is defined by the rule that each term is the sum of the one before it and the one after it. What is the 12th term?

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19. Consider three-dimensional chess, with a cubic or cuboidal 'chessgrid' of cells instead of a square or rectangular chessboard of squares. The knight's familiar  $\{2,1\}$ -L-shaped jump is now through faces of cells instead of over edges of squares. Suppose a knight occupies a cell in an empty chessgrid with infinite cells in all directions. To how many different cells may he move in one move?

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20. What is the minimum number of people that must be present in a room such that it is more likely than not that two people in the room share the same birthday? (*You may use a calculator, or better, a spreadsheet, for this one!*)