HSCP Maths Mash-up #5

No calculators, abaci, or props!

1. Pat's 101st birthday is tomorrow; his age in years will change from a square number to a prime number. How many times has this happened before in Pat's lifetime?

2. I write down a number with two digits. Then I write down the same two-digit number immediately next to it to form a number with four digits. What is the result of dividing the latter by the former?

3. For which regular polygon is the length of its *radius* (the distance from its centre to any of its vertices) equal to the length of any one of its edges?

4. The shortest street in the UK, *Ebeneezer Place*, in Wick, Scotland, is 2.06 metres long. The *Trans-Canada Highway*, one of the world's longest roads, is 7821 kilometres long. Roughly how many times longer than the street is the highway?

5. A small ink cartridge has enough ink to print 600 pages. Two medium cartridges print as many pages as three small cartridges, and two large cartridges print as many pages as three medium cartridges. How many pages can be printed using a large cartridge?

6. The perimeter of regular decagon P is eight times longer than the perimeter of regular octagon Q. Each edge of Q is 10 units long. How long is each edge of P?

7. What do you get if you divide the sum of the fifty lowest positive even numbers by the sum of the fifty lowest positive whole numbers?

8. What is the smallest prime number which is the sum of three different prime numbers?

9. In a triangle with interior angles of sizes x,y,z, measured in degrees, the mean average size of x and y is z. What is the value of z?

10. A solid wooden cube is painted blue on the outside. The cube is then dissected into eight identical smaller cubes. What proportion of the total surface area of these new cubes is blue?

11. The Kings of Clubs, Diamonds, Hearts, and Spades, and their respective Queens, are having an arm-wrestling competition! Everyone must wrestle everyone else, except that no King will wrestle his own Queen. How many wrestling bouts are there?

12. Foo thought of a positive whole number and multiplied it by either 5 or 6. Bar added 5 or 6 to Foo's answer. Finally, Qux subtracted either 5 or 6 from Bar's answer. Qux's result was 73. What integer did Foo think of?

13. A *space diagonal* is a diagonal (a line between non-adjacent corners) of a solid shape which does *not* lie on one of its faces, i.e., it runs through the shape (compare *face diagonal*). How many *space diagonals* has a cuboid? What is the length of any space diagonal of the unit cube (the cube with edges of length 1 unit)?

14. John-Boy has three times as many sisters as brothers. His sister Mary-Ellen has twice as many sisters as brothers. How many siblings has their family in total?

15. Which two positive integers, neither of which has a zero digit, multiply together to give 100000?

16. A regular octagon, with sides of length 1 unit, has its corners labelled P,Q,R,S,T,U,V,W, in clockwise or anticlockwise order. What are the precise lengths of the diagonal PS, PT, and PR?

17. What is the value of $1234567895^2 - 1234567890 \times 1234567900$?

18. A wedding photographer wishes to photograph the happy couple standing with four other married couples. But suppose, for one ridiculously hypothetical moment, that they all stand in line in a random order. What is the probability that everyone stands next to their own spouse by coincidence?

19. Suppose $x+(1/x) = 5$. What is the value of $x^2+(1/x^2)$?

20. In chess, the queen's move is through any number of squares in any single direction through the opposite edges (orthogonally) or vertices (diagonally) of those squares up to a capture or an edge of the chessboard. The *queen power* of a given square on a given chessboard is defined as the number of squares a queen may move to from that square in one move on an otherwise empty example of that board. For a given board, the queen power of a given square is solely dependent on the number of squares directly between it and the board edge closest to it, and hence regions of similar queen power form concentric square rings around the board centre. For a given square, let d be the number of squares directly between it and the board edge closest to it. On a standard (8-by-8) board, d takes one of the values 0,1,2,3. In terms of d, what is the queen power of any square on a standard board?