

## HSCP Maths Mash-up #10

No calculators, abaci, or props!

1. From midday to midnight, how many times are the minute hand and hour hand of an analogue clock at right angles to each other?

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2. A bag contains coins to the total value of £20.00. It contains only 50p, 20p, 10p, 5p, 2p, and 1p coins, and at least thirteen of each. What is the lowest possible number of coins in the bag?

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3. Sir Lance's very largest round table can fit sixty chairs around it. What is the highest number of people who can sit around the table such that each person is only sitting next to precisely one other person?

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4. Alice is 40 years old, and four times as old as Bob was when she was the same age as he is now. How old is Bob?

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5. Which of the statements below are true?
  - A. "Adding 1 to a number always makes it larger."
  - B. "Adding 1 to a number always makes it higher/greater."
  - C. "Subtracting 1 from a number always makes it smaller."
  - D. "Subtracting 1 from a number always makes it lower/lesser."
  - E. "Doubling a non-zero number always makes it higher/greater."
  - F. "Doubling a non-zero number always makes it larger."
  - G. "Halving a non-zero number always makes it lower/lesser."
  - H. "Halving a non-zero number always makes it smaller."

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6. Every day Ben has a breakfast, a lunch, and a dinner. The options for each meal are:  
Breakfast: Fruit and/or Cereal  
Lunch: Soup and/or Sandwich  
Dinner: Pasta and/or Curry  
One day, Ben eats four items. In how many different ways could he have done this?

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7. A rectangle has its corners labelled P,Q,R,S, in clockwise or anticlockwise order. Point A lies on side QR such that the ratio of lengths QA:AR is 6:5. What is the ratio of the area of region PQA to the area of region PARS?

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8. Six different prime numbers all less than 20 are separated into three pairs all having the same sum. What is the value of this sum?

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9. Alice, Carol, and Eve run in a 100-metre race. When Alice finishes, Carol is 16 metres behind, and when Carol finishes, Eve is 25 metres behind. The runners run at constant speeds throughout the race. When Alice finishes, how far behind is Eve?

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10. The order of digits is reversed in a specific two-digit positive whole number, giving a new number one less than half the original. What is the original number?

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11. What is the sum of all the powers of two from the sixth to the eleventh power?

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12. Points P and Q are connected by a straight line of length 24 units. The points A,B,C,D,E lie in order along PQ, with each not necessarily equidistant to its neighbours. PA, AB, BC, CD, DE, EQ are all edges of squares, with the squares of PA, BC, DE lying on the same side of PQ and the squares of AB, CD, EQ lying on the other side. A Logo turtle (*remember those?!*) is programmed to travel from P to Q via all five intermediate points but without any travel along PQ itself, which it does by a path along the edges of the squares. What is the length of that path (in units)?

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13. A 4-by-4 grid contains the letter C in the top-left cell, the letter L in all the cells of the bottom row and right-most column, and the letter O in all the remaining cells. Starting at the top-left cell, moving from one cell to the next either across a common edge or diagonally through a common corner, how many different routes spell "COOL"?

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14. At a chess conference, of those attending, 68% are fans of rapid chess and 39% are fans of blitz chess. Only 3% are fans of neither. What percentage are fans of both?

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15. A triangle has its corners labelled P,Q,R, in clockwise or anticlockwise order. The size of angle QPR is 40 degrees and the *internal bisectors* (straight lines which split each angle in half) of the angles at Q and R meet at point S (inside the triangle). What is the size of angle QSR?

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16. In a sequence of positive whole numbers, every term after the first two terms is the sum of the two previous terms in the sequence. If the fifth term is 2000, what is the greatest possible value of the first term?

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17. In Miss Venning's class, one third of the pupils bring a teddy bear to school. Last term, each boy took 12 books out of the school library, each girl took out 17 books, and each teddy bear took out 9 books. In total, 305 books were taken out. How many girls has Miss Venning's class?

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18. A cube is placed on a horizontal table. The corners of the top face are labelled A,B,C,D, in clockwise or anticlockwise order, and the corners of the bottom face are labelled P,Q,R,S such that P is directly below A, Q is directly below B, and so on. What is the size (in degrees) of angle ACS?

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19. In pencil, draw two straight guidelines, parallel to each other and not too far apart. On one of them, mark five points P,Q,R,S,T such that  $PQ = QR = RS = ST$ . Mark four points W,X,Y,Z on the other guideline where it is intersected by the *perpendicular bisectors* of PQ,QR,RS,ST respectively. Now, in pen, draw the following straight lines: WZ,WP,XQ,XR,YR,YS,ZT. Also, in pen, draw the reflection of these lines in the first guideline (PT). Finally, remove anything in pencil—guidelines and bisectors. How many hexagons has the resulting figure?

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20. In the previous Mash-up (#9), we considered so-called *orbits* of chessboards. To remind, two squares on a chessboard belong to the same orbit if the set of the four counts of squares between one of the given squares and each board edge in a line orthogonal to that edge is precisely the same set of the four counts as for the other given square. Now consider three-dimensional chess, with a cubical or cuboidal 'chessgrid' of cells instead of a square or rectangular chessboard of squares. The definition of an orbit extends naturally: two cells in a chessgrid belong to the same orbit if the set of the *six* counts of the cells between one of the given cells and each *grid face* in a line orthogonal to that face is precisely the same set of the six counts as for the other given cell. How many orbits has a *Raumschach* (5-by-5-by-5) chessgrid?