HSCP Maths Mash-up #11

No calculators, abaci, or props!

With the restrictions on daily life being eased ever-so-slightly for some last week, it is time for the Maths Mash-ups to have ever-so-slightly fewer questions! Hopefully sufficient remains herein to delight and to frustrate in equal measure all Masher-uppers feverishly dedicated, occasionally toe-dipping, and all inbetween!

1. Which number, spelt as a word, should replace X in the statement "There are X vowels in this short sentence."?

2. Imagine arranging matches end-to-end to make triangles. You must have more than two matches to do this, of course, but supposing this is true, with which precise number of matches does the task fail?

3. I start to read a 290-page book on a Sunday. I read four pages every day except on Sundays when I read 25 pages. How many days will it take for me to finish the book?

4. Alice, Bob, Carol, Dan, and Eve each toss a coin. At least four of the coins landed the same way up. In how many different ways could this result have been achieved?

5. A pentagon has its corners labelled P,Q,R,S,T in clockwise or anticlockwise order, and is presented with diagonals PR and PS. I wish to draw this figure without taking my pen off the paper and without going over any line more than once. At which corner, or corners, should I start and finish?

6. A recently-started community choir contained precisely 28 women and 13 men. Every week thereafter, four more women and seven more men joined the choir, and no-one left it. How many choristers were there in total in the week when the numbers of women and men in the choir were equal?

7. Sixty birds are perched in three trees. Ten minutes later, six birds had flown away from the first tree, eight had flown away from the second, and four had flown away from the third, leaving the same number of birds in each tree. How many birds were originally perched in the second tree?

8. Two rectangles and an equilateral triangle all have perimeters of the same length. One of the rectangles is a square with sides 9 units long. The other rectangle is non-square, with its longer edges having the same length as the edges of the equilateral triangle. What is the length of its shorter edges?

9. I choose a positive whole number, multiply it by four, then subtract thirty, then double it, and finally subtract ten. My answer has just two digits. What is the highest whole number I could have chosen?

10. Alice, Bob, Carol, Dan, and Eve are now sitting on a park bench, such that:
    Alice is not sitting on the far right,
    Bob is not sitting on the far left,
    Carol is not sitting at either end,
    Eve is not sitting next to Carol,
    Carol is not sitting next to Bob, and
    Dan is sitting to the right of Bob but not necessarily next to him.
    Who is sitting on the far right of the bench?

11. Two identical squares, one with corners PQRS, labelled clockwise, and the other with corners PTUV, labelled anticlockwise, share corner P and are turned relative to each other such that the size of angle PQT is 70 degrees. What is the size of angle SPV?

12. What is the lowest number of identical cubes required to fill a box measuring 56 by 40 by 16 units?

13. There are fifteen composite numbers (non-prime numbers greater than 1) less than 50 which are not divisible by a square number greater than 1, but how many of these are also not divisible by a square number greater than 1 *when they are doubled*?

14. "abab"×"ccc" = 639027, where a,b,c are digits. What is the value of $a+b+c$?

15. Marbles of diameter 1 unit are dropped into a vertically-orientated cylindrical tube with a closed bottom. The marbles pack under gravity to form two vertical 'columns' of marbles, intersecting in the horizontal cross-section, and such that, in each column, each marble is touching the marble immediately beneath it. What is the internal diameter of the tube?

16. Let n-sudoku be the puzzle game in which a solution consists of n^2 different symbols filling an n^2-by-n^2 grid, partitioned into n^2 n-by-n sub-grids, such that no row, column, nor sub-grid contains a particular symbol more than once. The usual game is, of course, 3-sudoku, which uses a nine-by-nine grid and nine symbols, usually (and entirely arbitrarily) the digits 1 to 9. This question concerns the simpler game of 2-sudoku, which uses a four-by-four grid and four symbols.
    Consider the patterns *a single symbol* could present on a 2-sudoku grid—not configurations of a completed grid, but just of one symbol within it. Let two such patterns be in the same *pattern class* if they are the same pattern under a symmetrical rotation or reflection of the grid. How many different patterns are possible altogether? How many different pattern classes do they form?

    Postscript. Which approach do you think you would prefer here: to find all the patterns, then to consider the symmetries of the grid to categorise them into pattern classes, or to find an example pattern of each pattern class, then to consider the symmetries of the grid acting on the example patterns to find all the patterns?