

HSCP Maths Mash-up #12

No calculators, abaci, or props!

1. Alice, Bob, Carol, and Dan are queuing for the bus. In how many different ways can they line up in single file without Dan being the last in the queue?
2. A river 120 metres wide is spanned by a bridge, of which one quarter is over one bank of the river and one third is over the other bank. How long (in metres) is the bridge?
3. I must choose seven different positive whole numbers together having a mean average of 7. What is the greatest possible such number I could choose?
4. Of all the numbers that could be produced by changing any one digit of the number 200, how many are prime?
5. At a holiday camp, the ratio of boys to girls is 3:4 and the ratio of girls to adults is 5:7. What is the fully simplified ratio of children to adults at the camp?
6. A black cube has blue cubes of the same size glued face-to-face to each of its six faces to form a three-dimensional "cross". If red cubes of the same size are now glued face-to-face to all of the spare faces of this cross, how many red cubes are required?
7. Alice is thinking of three positive whole numbers, not necessarily all different. Alice tells Bob that their product is 36, and also reveals their sum, but Bob cannot tell what the three numbers are. What is the sum of Alice's three numbers?
8. An archery target has four regions, each scoring 0, 2, 3, and 6. Two arrows are shot and hit the target. How many different totals could be scored?
9. For how many values of n are both $n/2$ and $2n$ three-digit positive whole numbers?
10. What is the edge length (in units) of a cube whose volume in cubic units is the same as its surface area in square units?
11. The *product of powers of primes* representation of the number 7056 is $2^4 \times 3^2 \times 7^2$. What is the square root of 7056?
12. How many three-digit positive whole numbers have an odd number of positive factors?
13. A parallelogram has its corners labelled W,X,Y,Z, in clockwise or anticlockwise order. It is partitioned as nine smaller parallelograms, all having the same angles, by four straight lines, two parallel to WX and two parallel to WZ. The perimeters of the middle-top, middle-right, middle-bottom, and middle-left parallelograms are respectively 4, 5, 8, and 11 units. The perimeter of WXYZ itself is 21 units. What is the perimeter of the central parallelogram?
14. When the digit 1 is *appended* to a certain five-digit whole number, the resulting six-digit number is precisely three times as great as the one which results when the digit 1 is instead *prepended* to it. What is the five-digit number?
15. Three unit circles (circles of radius 1 unit) are arranged tangentially to each other (each circle touches the other two without overlapping). What is the radius of the smallest possible circle which encloses all three of them fully?

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16. Let k -latin-square be the puzzle game in which a solution consists of k different symbols filling a k -by- k grid such that no row nor column contains a particular symbol more than once. In other words, latin-square is sudoku without the sub-grids, and n^2 -latin-square is a generalisation of n -sudoku, using a grid of the same size. This question concerns the game of 4-latin-square, which is a generalisation of 2-sudoku.

Consider the patterns a *single symbol* could present on a 4-latin-square grid—not configurations of a completed grid, but just of one symbol within it. Let two such patterns be in the same *pattern class* if they are the same pattern under a symmetrical rotation or reflection of the grid. How many different patterns are possible altogether, and how many pattern classes do they categorise into?

Postscript. This question is, of course, an extension to Q16 of the previous Mash-up (#11). The postscript in that question mentions opposite approaches: patterns-into-classes and classes-determining-patterns. If you attempted that question, will you switch or keep your approach for this question?
