HSCP Maths Mash-up #13

No calculators, abaci, or props!

1. In four years' time Alice will be three times as old as she was two years ago. How old will Alice be in one year's time?

2. The kettle in my kitchen is 80% full of water. After pouring out 20% of the water in it, 1152 millilitres remains. What volume does my kettle hold when it is full?

3. In your pocket are eight watermelon jellybeans, four vanilla jellybeans, and six raspberry jellybeans. What is the least number of jellybeans you must take out of your pocket to be certain that you take at least one of each flavour?

4. For every two-digit positive whole number, subtract the units digit from the tens digit. What is the sum of all the results?

5. Over two days, Alice and Bob take and eat chocolates from a box of 21 chocolates. On Monday, Alice takes two-thirds of the number of chocolates that Bob takes. On Tuesday, Alice takes one half of the number of chocolates that Bob takes, after which the box is empty. How many chocolates does Alice scoff in total?

6. Four of Usain's under-11s track club formed a squad to run a 4x100m relay. The runner of the final leg was also the fastest, in 15 seconds. Second leg took 1 second longer than first leg, third leg took 10/9 as long as second leg, and final leg took 3/4 as long as third leg. What time (in seconds) did the squad post?

7. A square is drawn on a piece of card and divided up into sixteen smaller squares as a four-by-four grid. The figure is then cut into two identical pieces along either of the two diagonals of the original square. How many triangles, and how many rectangles, are on a single piece?

8. To break into a safe, you must find the correct three-number combination for a 20-way combination lock. Each number is an integer from 1 to 20. However, you have some information to help: the first number is a square number, the second number is an odd number, and the third number is a prime number. With this information, what is the percentage decrease in the number of possible combinations?

9. Rectangle P is 8 by 10 units and rectangle Q is 9 by 12 units. They overlap, and the area of P not overlapping Q is 37 square units. What is the area of Q not overlapping P?

10. A railway line has stations A,B,C,D,E in order. At D, there is a branch line to F,G,H,J. At G, there is a branch line to K,L,M. Travelling only by train, starting at any station, finishing at any station, and with no restrictions on the route taken, what is the least number of stations which must be visited more than once in order to visit all the stations?

11. In the equation "A"+"A"+"BB" = "CCC", the letters A,B,C represent different digits. Which digit does A represent?

12. Points P,Q,R,S lie in order along a horizontal straight line, with $PQ = QR = RS = 2$ units. Semicircles with diameters PQ and RS lie above the line, and semicircles with diameters PS and QR lie below the line. The precise area of the closed shape formed by the semicircles is $p \times \pi$ square units. What is the value of p?

13. Ten square tiles, identical apart from their numbering from 1 to 10, are arranged edge-to-edge and in three rows as follows: 1,2,3,4 (from left to right) in the top row, 5,6,7,8,9 in the second row, with 5 being directly underneath 2, and just 10 in the bottom row, directly underneath 5. You must remove one tile from the figure without changing the length of its perimeter. Which tile, or tiles, could you pick?

14. In a pencil case are nine pencils. At least one of the pencils is blue. In any group of four pencils, at least two are the same colour. In any group of five pencils, at most three are the same colour. How many of the pencils are blue?

15. How many three-digit positive whole numbers are equal to 34 times the sum of their own digits?

16. When a piece of A4-size paper is folded precisely in half parallel to its short edge, it results in A5-size. An-size has the special property that it represents a *similar* rectangle to A($n-1$)-size but having precisely half the area. (In geometry, "similar" means the very same shape, with corresponding angles equal and ratios of lengths of corresponding edges equal, but not necessarily the same size, so not necessarily lengths of corresponding edges themselves equal.) For this special property to hold true, the lengths of the long and short edges must be in a special ratio. If the short edge is 1 unit long, precisely how long is the long edge?