HSCP Maths Mash-up #15

No calculators, abaci, or props!

1. On a digital clock displaying hours, minutes, and seconds, how many times in each 24-hour period do all six digits change simultaneously?

2. If all the whole numbers from 1 to 1000 are written down, which digit appears the lowest number of times?

3. A mouse and a grouse cost £90 in total. Three mice and two grice cost £210 in total. How much (in £s) does a grouse cost?

4. Alice, Bob, Carol, and Dan hold the top four rankings in a local tennis league. The sum of the rankings of Alice, Bob, and Dan is six. The sum of the rankings of Bob and Carol is also six. Bob is ranked "higher" than Alice. Who's in top spot?

5. For her latest modernist art installation, Helen has fixed a 2-by-2 feet square sheet of steel to a wall, and she has four 1-by-1 foot square magnetic tiles to fit to the sheet such that none of the sheet is visible and the lines separating the tiles are unnoticeable. The tiles are plainly coloured; two are the same shade of green, one is red, and the other is blue. How many different-looking installations can Helen create?

6. Jake dances clockwise around the maypole, making one revolution every five seconds. Starting from a point diametrically opposite Jake's starting point, Shea dances anticlockwise, making one revolution every six seconds. How many times do they pass each other in the first minute?

7. Sue has a cuboidal block of fudge measuring 2 by 3 by 6 inches. She wishes to cut the whole block up into cubes whose edge lengths are whole numbers of inches. What is the least number of cubes she can obtain?

8. Twelve squares are joined together as a grid of four columns by three rows. How many different figures are possible if precisely four of the squares of the grid are shaded such that the resulting figure has two lines of symmetry?

9. I was born in 1978, a year with a special property: when you add the number formed by the first two digits to the number formed by the last two digits, the result is equal to the number formed by the middle two digits ($19+78 = 97$). Which is the next year to have this special property?

10. A list of ten numbers contains two of each of the numbers 0,1,2,3,4. The two 0s are next to each other, the two 1s are separated by one number, the two 2s by two numbers, the two 3s by three numbers, and the two 4s by four numbers. The list starts "3,4,...". What is the last number in the list?

11. What is the lowest multiple of 15 all of whose digits are 0s and 1s?

12. Three identical squares are arranged such that one side of each square forms a side of a central triangle, and each corner not shared with the central triangle is joined by a straight line to the corner 'visible' to it of the neighbouring square, giving three further triangles. What is the size (in degrees) of an *acute* angle in any of these three triangles?

13. The nth term of a sequence is $(1/2)^n$. What is the difference between the fifth and eighth terms of the sequence?

14. Box P has p chocolates and box Q has q chocolates, where p and q are either both even numbers or both odd numbers, and p is greater than q. In terms of p and q, what is the lowest whole number of chocolates which would have to be moved from P to Q so that Q has more than P?

15. A circle both *inscribes* a square (the circle touches, but does not cross, each side of the square) and *circumscribes* another square (each corner of this square lies on the circle). What is the ratio of the area of the large square to the area of the small square?

16. Two ponds each contain frogs and toads. In one pond, the ratio of frogs to toads is 3:4. In the other pond, the ratio of frogs to toads is 5:6. The total number of frogs is 36. What is the highest possible total number of toads?