HSCP Maths Mash-up #6
No calculators, abaci, or props!

1. What is the smallest possible difference between two different nine-digit whole numbers, each of which includes all of the digits 1 to 9?

2. Two 30-centimetre rules are placed together such that the 7 cm mark on each is aligned with the 10 cm mark on the other. What is the overall length of this arrangement, the distance between the far ends of the rules?

3. I plant sixty tulip bulbs. On flowering, half are yellow, one third of those which are not yellow are red, and one quarter of those which are neither yellow nor red are pink. The remainder are white. What proportion of the tulips are white?

4. The numbers 1,2,3,4,5 are placed at the corners of a pentagon, one per corner, such that the numbers at neighbouring corners always differ by more than one. What is the result of multiplying the two numbers neighbouring number 5?

5. It's student president election time. A candidate receiving more votes than any other wins. The four candidates receive 83 votes between them. What is the lowest number of votes the outright winner could receive?

6. A car's odometer, in miles, shows the number 15951, which is a *palindromic number* (like a palindrome, a word spelled the same way backwards as forwards). The next time a palindromic number appears on the odometer is precisely two hours later. What is the average speed (in miles per hour) of the car during these two hours?

7. From noon until midnight Clever Cat sleeps under the old oak tree, and from midnight until noon he is awake telling stories. A poster above him on the tree says "Two hours ago, Clever Cat was doing the same thing as he will be doing in one hour's time." For how many hours per day does the poster tell the truth?

8. The geometric sculpture Cubo Vazado ("Emptied Cube"), by the Brazilian artist Franz Weissmann, can be conceptualised as the removal of small cubical blocks from a larger solid cube. Find a photograph of it, or any of its copies, online (search for "Weissman Cubo Vazado"). If its longest edges have length 3 units, its shortest edges have length 1 unit, and all of its other edges have length 2 units, what is the volume (in cube units) of the sculpture?

9. A retailer pays a wholesaler £30 for a specific item. It wishes to sell the item with a 10% discount on the tag price but still to make 20% profit on the cost price. What price should go on the tag?

10. A water tank is connected to two pumps. One of the pumps can fill the tank by itself in precisely two hours. The other can fill the tank by itself in precisely three hours. How long (in minutes) does it take to fill the tank when both pumps are used simultaneously?

11. Which regular polygon has twice as many diagonals as it has sides? (To remind, a diagonal is a line between non-neighbouring corners.)

12. How many *space diagonals* has a *regular octahedron*, the convex regular solid shape ("Platonic solid") which looks like two square pyramids joined base-to-base? (To remind, a space diagonal is a diagonal of a solid shape which does not lie on one of its faces.) What is the length of any space diagonal of a regular octahedron with edges of length 1 unit?

13. In terms of n, what is the maximum possible number of *reflex* interior angles (interior angles greater than 180 degrees) in a closed polygon with n sides?

14. What is the sum of all the numbers less than 120 which are each the product of precisely three different prime numbers?

15. I write down a positive whole number with n digits. Then I write down the same n-digit whole number immediately next to itself to form a positive whole number with $2n$ digits. In terms of n, what is the result of dividing the latter by the former?

16. Six pieces of string are clamped in a vice, with the ends of each of the strings sticking out above and below the vice. I tie all the upper ends together in pairs, then do the same with the lower ends. If I tie the pairs randomly, what is the probability that all six pieces of string will form a single loop?

17. In a sequence of seven terms, all the terms are positive single-digit numbers. Every mini-sequence of four consecutive terms adds up to 16, and every mini-sequence of five consecutive terms adds up to 19. What is the sum of all seven numbers?

18. A square has its corners labelled P,Q,R,S, in clockwise or anticlockwise order. It encloses two identical circular arcs which both connect the midpoints of edges QR and SP, but one with centre the midpoint of PQ and the other with centre the midpoint of RS. What proportion of the area of the square is enclosed between the two arcs?

19. Given an *indefinitely long* list of positive whole numbers 1,2,3,..., what fraction best approximates the proportion of the list which are *not* multiples of either 2, 3, or 7?

20. Consider three-dimensional chess, with a cubic or cuboidal 'chessgrid' of cells instead of a square or rectangular chessboard of squares. The queen's move is now through any number of cells in any single direction through the opposite faces ('fully orthogonally'), edges ('semi-orthogonally'/'semi-diagonally'), or vertices ('fully diagonally') of those cells up to a capture or a face of the grid. Suppose a queen occupies a cell in an empty chessgrid with infinite cells in all directions. In how many *different directions* may she move in one move?