

HSCP Maths Mash-up #7

No calculators, abaci, or props!

1. Half-way through the second quarter of a netball match, what proportion of the match remains to be played?

2. After tennis training, Roger collects twice as many balls as Novak and five more than Rafael. They collect 35 balls in total. How many balls does Roger collect?

3. Alice, Bob, Carol, and Dan have their 10th, 12th, 15th, and 19th birthdays today. In how many years' time will their ages first sum to 100?

4. An infinitely long word which repeats every seven letters begins "ABRACAD...". What are the 1000th and 1001st letters in the word?

5. Seven identical circular coins are laid together, but not overlapping, on a flat surface to form a regularly hexagonal figure. A regularly hexagonal frame precisely encloses the coins, such that each coin is unable to slide in any direction on the surface. What is the largest number of coins that may be removed one-by-one such that, with each removal, every remaining coin is still unable to slide?

6. The sock drawer contains ten identical red socks, eight identical blue socks, and four identical green socks. If socks are taken from the drawer at random, what is the smallest number of socks which must be taken to be sure of *two* pairs of matching socks?

7. Six unit squares (squares with edges of length 1 unit) are arranged together in a 3-2-1 tower with vertical symmetry. Two diagonals connect the midpoint of the top edge of the top square to the bottom-right and bottom-left corners of the tower respectively, forming a triangle between those three points. What is the area (in square units) of that triangle?

8. The distance between Exeter and London is 175 miles. One day, Alice left Exeter for London at 10:00, and Bob left London for Exeter at 13:00 on the same route in reverse. Up to the moment when they crossed over, Alice's average speed was 25 miles per hour and Bob's was 35 mph. At what time did Alice and Bob cross over?

9. Which of the statements below are true?
 - A. "Doubling a positive number always makes it larger."
 - B. "Squaring a positive number always makes it larger."
 - C. "Taking the square root of a positive number always makes it smaller."

10. What proportion of the whole numbers from 1 to 8000 are cube numbers?

11. A square is folded precisely in half and then precisely in half again. Across all of the different possible shapes which could result, how many right angles are there in total?

12. A square and a triangle share an edge, forming an irregular pentagon. The perimeters of the square and the triangle are the same length. What is the ratio of the length of the perimeter of the pentagon to the length of the perimeter of the square?

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13. The following four statements are about a positive whole number N . Precisely two of them are true (and the other two are false):
- “ N is a multiple of 6.”
 - “ N is a multiple of 15.”
 - “ N is a multiple of 30.”
 - “ N is less than 12.”
- What is the value of N ?
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14. Six dominoes—[1|2], [1|5], [2|5], [4|5], [4|6], [5|6]—are to be arranged in a hexagonal ring, with the pips matching for every pair of neighbouring dominoes, as usual. In a completed ring, which of the other five dominoes definitely *cannot* be placed next to [1|5]?
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15. I choose two different single-digit positive whole numbers at random, then calculate their sum. What is the probability that the sum is a single-digit number?
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16. A cuboid of dimensions 14 by 7 by 5 units is filled completely with unit cubes (cubes with edges of length 1 unit). How many of the unit cubes touch a face of the cuboid?
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17. A jar contains red marbles, blue marbles, and green marbles. All of the marbles are red except for twelve, all of them are blue except for fourteen, and all of them are green except for sixteen. How many marbles are in the jar?
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18. Consider the eight possible products of the two whole numbers “ mn ” and “ np ” where m, n, p are digits such that $m+1 = n = p-1$. Only one of these products is divisible by all of the whole numbers from 1 to 10. For this product, what is the value of n ?
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19. Seven cards are numbered from 1 to 7 and placed in a box. Alice takes three cards from the box, then Bob takes two cards, leaving two cards in the box. Alice looks at her cards, then tells Bob “I know that the sum of the numbers on your cards is even.” What is the sum of the numbers on Alice’s cards?
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20. A sequence of black-and-white chessboard figures begins with the trivial 1-by-1 black square. Every next figure in the sequence is a chessboard figure with one more black square in each side than the previous figure (consequently, all the corner squares of every figure are black, and the overall shape of every figure is square). How many black squares are in the fifteenth figure in the sequence?
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