HSCP Maths Mash-up #8

No calculators, abaci, or props!

1. How many different cubes have three faces coloured red and three faces coloured blue?

2. In the expression "1#2#3#4", each "#" is to be replaced by either add or multiply. What is the largest value of all the expressions which can be obtained in this way?

3. No more than twelve people can sit, evenly spaced, around a large square table. Sir Lance arranges eight of these square tables in a row to make one long rectangular table. What is the maximum number of people that can sit evenly spaced around this long table?

4. Granny spent one third of her weekly pension last Thursday evening at Wings Bingo, and one quarter of what remained on Friday night at Studio 54. What proportion of the original amount was left for her big night out on Saturday (at the Deadmau5 gig)?

5. Four identical equilateral triangles, two shaded red and the other two blue, are arranged to make one large equilateral triangle. How many arrangements are possible?

6. How many digits has the correct answer to the calculation 123,123,123,123 ÷ 123?

7. The Grand Old Duke of York lost 10% of his battalion as he marched them up to the top of the hill, and he lost 15% of the rest as he marched them down again. What percentage of his original battalion remained?

8. Alice, Bob, and Carol went to the lido fifteen times last summer. Alice paid for everyone eight times and Bob paid for everyone seven times. At the end of the summer, Carol owed £30. How should Carol split this between Alice and Bob so that each has paid the same amount?

9. The sum of ten consecutive whole numbers is 5. What is the largest of these numbers?

10. There are six different three-digit positive whole numbers each of which contains all the digits 1, 3, and 5. How many of these numbers are prime?

11. A square is divided up into three 'parallel' congruent rectangles. The middle rectangle is removed, rotated by a quarter-turn, and placed against the side of the original square to form an irregular octagon. What is the ratio of the length of the perimeter of the original square to the length of the perimeter of the octagon?

12. A cuboid with a pair of square opposite faces is presented. The area of each of the square faces is one quarter of the area of each of the other four faces. The total surface area of the cuboid is 72 square units. What is the area of any one of the "other" four faces?

13. Each of the fractions 2637/18459 and 5274/36918 uses each of the digits 1 to 9 precisely once. The first fraction simplifies to 1/7. What does the second fraction simplify to?

14. What is 50% of 18.3 plus 18.3% of 50?

15. A *multiplication grid* is a grid in which each cell contains the product of the number in its respective row header with the number in its respective column header. The body (the cells without the headers) of this particular multiplication grid is 5-by-5, and its cells are hidden except for the following:
cell r1c2 = 10 (counting rows from top to bottom and columns from left to right),
r1c4 = 20, r2c1 = 15, r2c3 = 40, r3c1 = 18, r3c4 = 60, r4c2 = 20, r4c5 = 24, r5c3 = 56.
What is the sum of all the cells in the *leading diagonal* (r1c1, r2c2, r3c3, r4c4, r5c5)?

16. Compare the sums of the first n cube numbers, for n = 1,2,3,4,5,6 (so, for $n = 1$, the required sum is 1^3; for $n = 2$, it is 1^3+2^3, etc.). How many of these six sums are square numbers?

17. A regular pentagon has its corners labelled P,Q,R,S,T in clockwise or anti-clockwise order. What is the size (in degrees) of angle RPS (the angle between diagonals RP and PS)?

18. In a group of 48 children, the ratio of boys to girls is 3:5. How many boys must join the group to make the ratio of boys to girls 5:3?

19. An octagon has its corners labelled 1 to 8 and every corner connected to every other. Each connection is labelled with the sum of the numbers at its two end-points. How many of these labels are multiples of 3?

20. There are thirty students in my maths tutoring practice. Twenty of them like informatics and eighteen of them like physics. Twice as many students like both subjects as like neither of them. How many of my students like informatics but not physics?