HSCP Maths Mash-up #9

No calculators, abaci, or props!

1. At what time in the morning does the hour hand of an analogue clock point directly at the 38-minute mark?

2. What is the value of $1-2+3-4+5-6+\ldots+999-1000+1001$?

3. I spend one fifth of the cash in my wallet, then one fifth of what remains in it. If I spent £72 in total, how much cash was in my wallet to start with?

4. In two years' time, Carol's brother Bob will be twice as old as he was two years ago, and in three years' time her sister Alice will be thrice as old as she was three years ago. How many years older (or younger) is Alice than Bob?

5. Two identical cuboid-shaped bricks, each measuring 3 by 1 by 1 units, are laid end-to-end, and a third brick identical to the others laid in parallel on top of them. The resulting stack has three orthogonal profiles: top, side, and end-on. What is the length of the perimeter of the side profile?

6. A hotel has rooms on six floors, the floors being numbered 1 to 6. It has 36 rooms on each floor, numbered "*n*01" to "*n*36", where the digit *n* is the number of the respective floor. In numbering all the rooms, how many times will the digit 3 be used?

7. In how many different ways can you split a team of four people into two separate teams such that there is at least one person in each team?

8. In the window of a cycle shop are some unicycles, some bicycles, and some tricycles. In total, there are seven saddles and thirteen wheels, and there are more bicycles than tricycles. How many unicycles are in the window?

9. An older television, with aspect ratio 4:3, is set to display the entire image of a widescreen broadcast, having aspect ratio 16:9, using as much of the display as possible but without distorting the image. What proportion of the display is *not* covered by the image?

10. The sum of three different prime numbers is 40. What is the difference between the two biggest of these three numbers?

11. Each side of an isosceles triangle is a whole number of units long. Its perimeter is 20 units long. How many different such triangles are possible?

12. An *exterior angle* of a corner of a polygon is the angle turned by a pen on its journey around the corner (the angle between the neighbouring side and the "current side extended"). A triangle is drawn for which the sizes of its *exterior* angles are in the ratio 4:5:6. In what fully simplified ratio are the sizes of its respective *interior* angles?

13. Consider the cipher in which A=1, B=2, C=3, etc., and words are encoded as single numbers by multiplying together the values of their letters. In what ratio, fully simplified, is the code for the word "TRIANGLE" to the code for the word "RECTANGLE"?

14. The cells of a 4-by-4 grid are filled as follows, from left to right and top to bottom: 9,6,3,16,4,13,10,5,14,1,8,11,7,12,15,2. Two of the numbers in the grid can be swapped to turn it into a *magic square*. Which two numbers?
[In a magic square, every row, column, and each of the two main diagonals has the same sum, called the *magic sum*.]

15. A caterpillar starts from its hole and moves across the ground, turning a right angle either left or right after each hour. It moves 2 metres in the first hour, 3 m in the second hour, 4 m in the third hour, and so on. What is the greatest distance it can be from its hole after seven hours?

16. In the product $2\times$"SEE" $=$ "AXES", each different letter represents a different non-zero digit. Which digit does X represent?

17. Consider a cuboid with every face painted a different colour. In how many different ways can you make a *planar* ('uncurved') slice through the cuboid without slicing any of its edges into two parts?
[Note that you are allowed to slice *along* an edge—this is not the same thing as slicing an edge into two parts.]

18. Every one of n nodes is connected by one arc to each and every other node. In terms of n, how many arcs are there?

19. Consider two squares on a chessboard to belong to the same *orbit* if the four counts of squares between one of the given squares and each board edge in a line orthogonal to that edge are precisely the same four counts as for the other given square. By this definition, how many orbits has a standard (8-by-8) chessboard?
[To understand the above definition of *orbit*, it can help to imagine all the squares on a chessboard being the same colour, then to consider the rotational and reflective symmetry of the board. Together, all the orbits of a chessboard *partition* it, i.e., every square is in precisely one orbit. Consequently, no orbits overlap, and the total number of squares in all the orbits is simply the total number of squares on the board. You may even have been using orbits already, without necessarily knowing the term, to solve quicker some of the chessboard problems in previous Mash-ups.]

20. Which triples (a,b,c) solve the equation $b\times c + c\times a + a\times b = a\times b\times c$, where a,b,c are positive whole numbers? How many such triples are there in total?
[To get maximum smarties on this one, you should be able to show that there are no more solution triples than the ones you find. Guesswork/intuition may find some or all of the solution triples, but it won't tell you when to stop searching!]