HSCP Maths Mash-up #17

No calculators, abaci, or props!

1. A million seconds is how many days, hours, minutes, and seconds, in simplest terms?

2. Let S be the set of all the whole numbers from 1 to 100. What is the result if the sum of all odd members of S is subtracted from the sum of all even members of S?

3. Ben glugs 1/5 of a bottle of water, and later another 3/10 of the water remaining. What proportion remains after that?

4. For his annual wage, a medieval apprentice was promised one hundred florins and a cloak. But he left the apprenticeship after only seven months, for which his due was twenty florins and the cloak. How many florins was the cloak worth?

5. Alice and Bob played a two-player game many times over. For each game, a winner is always determined. The winner is awarded two points and the loser one point. Alice won precisely four games and Bob had a final score of ten points. How many games did they play?

6. Four copies of the right-angled triangle with side lengths 5,12,13 units are joined together, without gaps or overlaps, to make a parallelogram. What is difference (in units) between the highest and lowest possible lengths of the perimeter of the parallelogram?

7. Helen always cycles to work by the same route. When she averages 15 kilometres per hour, she arrives ten minutes late. When she averages 30 km/h, she arrives ten minutes early. What speed (in km/h) should she average to arrive on time?

8. As a fully simplified single fraction, what is the value of (1/3)+(1/4)+(1/5)+(1/9)+(1/12)+(1/15)?

9. The size of some of the interior angles of an irregular octagon is 120 degrees. The size of all of the other interior angles is 160 degrees. How many of the interior angles have size 120 degrees?

10. A robotic ant is initially facing due north on a large map laid flat. It is programmed to make repeated moves as follows: travel 5 centimeters forward in a straight line, then turn clockwise through an angle which increases by 10 degrees with each move. On the first move, the angle turned is 10 degrees. How far (in centimeters) has the ant travelled by the time it is first facing due east on the map?

11. What pair of whole numbers (n,m) satisfies the equation
$(\sqrt{2})^1+(\sqrt{2})^2+(\sqrt{2})^3+(\sqrt{2})^4+(\sqrt{2})^5 = n+(m\times\sqrt{2})$?

12. How many *planes of symmetry* has a plain cube?
(If it helps, it is the same number as for a plain regular octahedron!)

13. A square is divided up into nine small squares in a three-by-three grid. Each small square can be coloured completely black or completely white. What is the product of the lowest and highest numbers of small squares which can be coloured black such that the design created has two-fold rotational symmetry but no lines of symmetry?

14. In the equation 19×(A+B+B) = “ABB”, the letters A and B represent different digits. What is the highest whole number which can be represented by “ABB”?

15. Ten boxes are arranged in a 4-3-2-1 tower with vertical symmetry. The bottom-left and bottom-right boxes respectively contain the numbers 12 and 78. The middle box in the row-of-3 contains the number 90. The number in each box is obtained by adding the two numbers in the two boxes immediately underneath it. What number is contained in the top box?

16. Two rectangles, both measuring 12 by 8 units, are each cut along one diagonal. The four resulting triangles are arranged medium-edge-to-short-edge, without overlapping, to form a square with a square gap in the middle (such that the figure has four-fold rotational symmetry but no lines of symmetry). What is the ratio of the area of the large square to the area of the small square?